Lecture 3: Magnetization, cables and ac losses

Magnetization

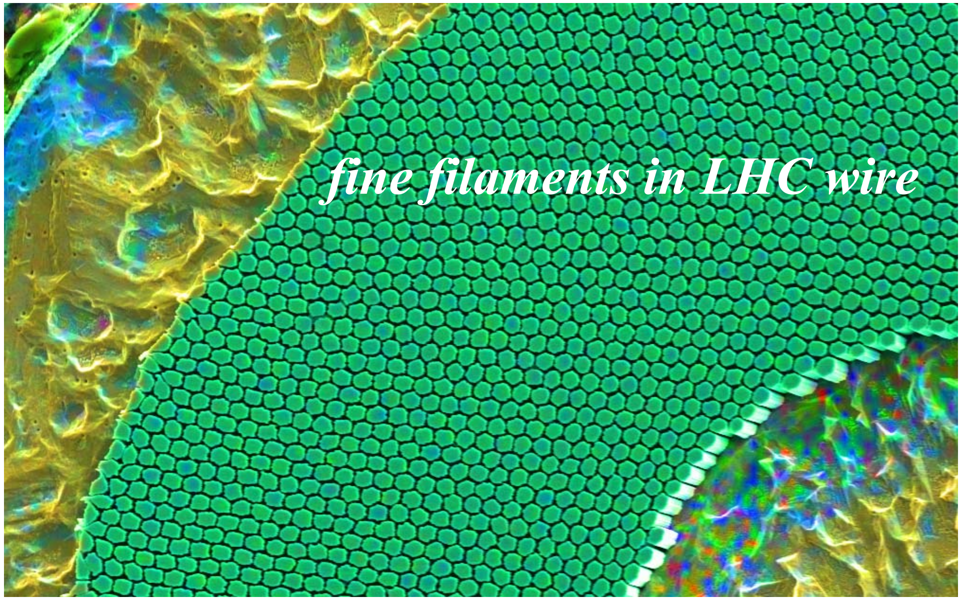
- magnetization of filaments
- coupling between filaments

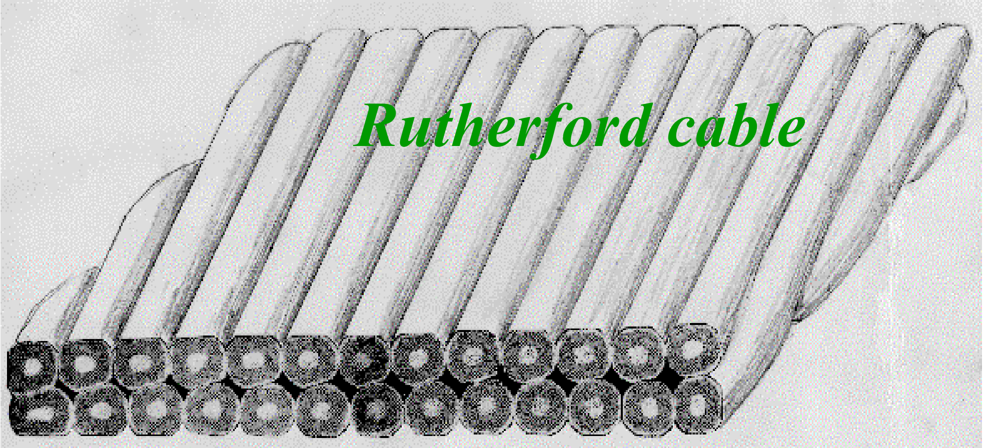
Cables

- why cables?
- coupling in cables
- effect on field error in magnets

AC losses

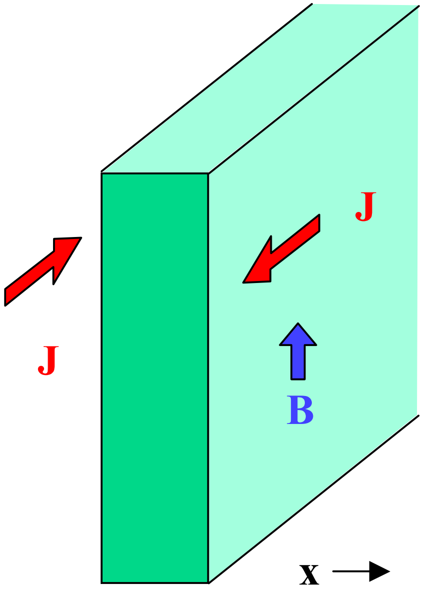
- general expression
- losses within filaments
- losses from coupling


fine filaments in LHC wire


Rutherford cable

Persistent screening currents

- ***screening currents*** are in addition to the ***transport current***, which comes from the power supply
- like eddy currents but, because no resistance, they don't decay



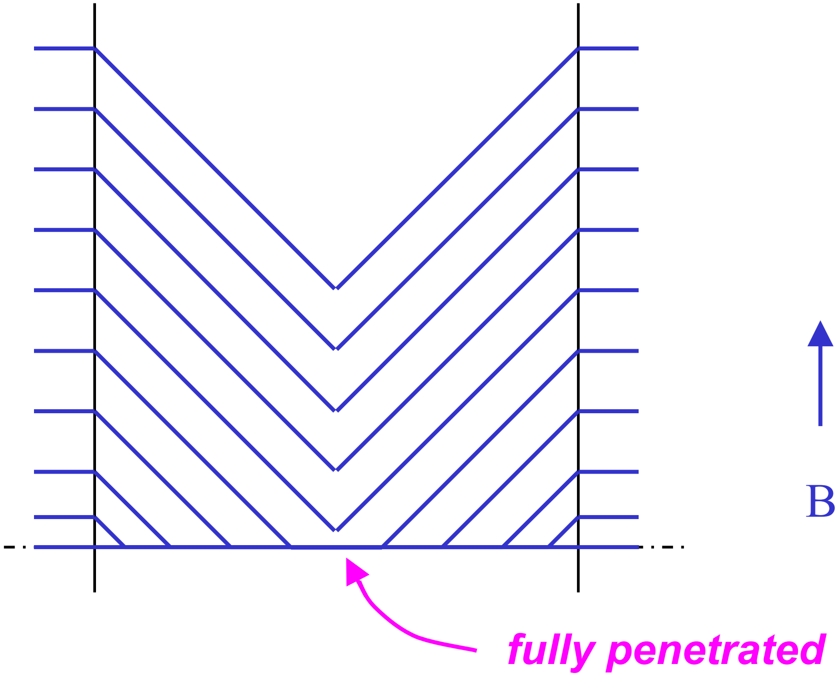
- ***dB/dt*** induces an electric field ***E*** which drives the screening current up to critical current density J_c
- so we have $J = +J_c$ or $J = -J_c$ or $J = 0$ nothing else
- known as the ***critical state model*** or ***Bean model***
- in the 1 dim infinite slab geometry, Maxwell's equation says

$$\frac{\partial B_y}{\partial x} = -\mu_o J_z = \mu_o J_c$$

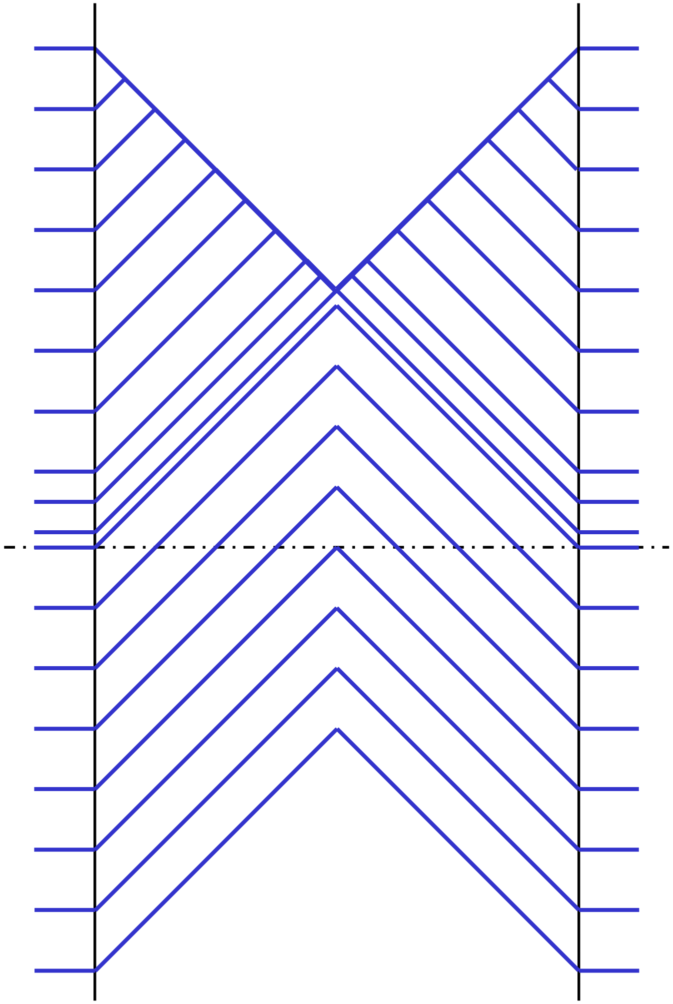
- so uniform J_c means a constant field gradient inside the superconductor

The flux penetration process

plot field profile across the slab



field increasing from zero



field decreasing through zero

Bean critical state model

- **current density everywhere is ±J_c or zero**
- **change comes in from the outer surface**

Magnetization of the Superconductor

When viewed from outside the sample, the persistent currents produce a magnetic moment.

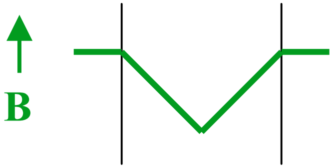
Problem for accelerators because it spoils the precise field shape

We can define a magnetization (magnetic moment per unit volume)

$$M = \sum_V \frac{I.A}{V}$$

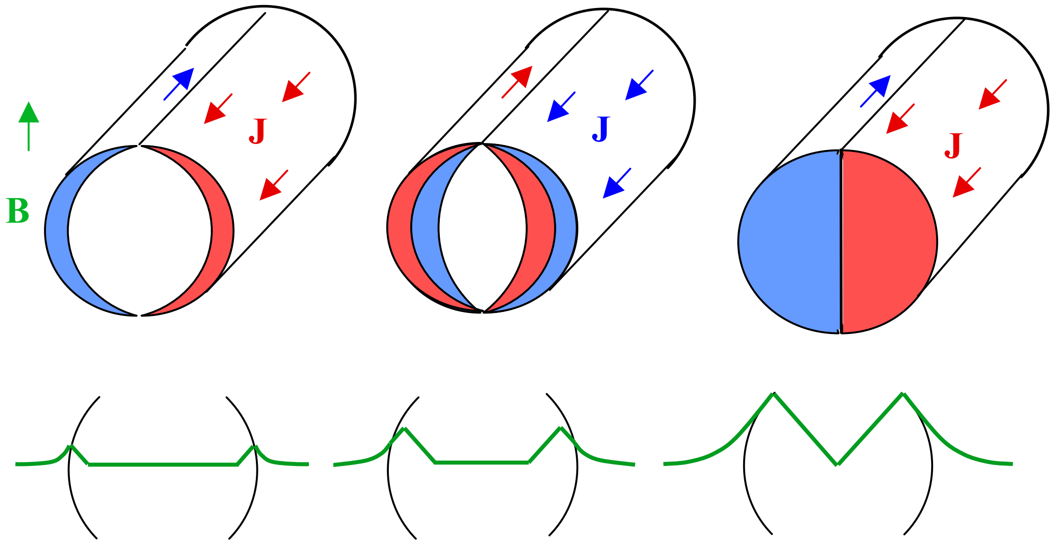
NB units of H

for a fully penetrated slab



$$M = \frac{1}{a}\int_0^a J_c.x.dx = \frac{J_c.a}{2}$$

for **cylindrical** filaments the inner current boundary is roughly elliptical (controversial)

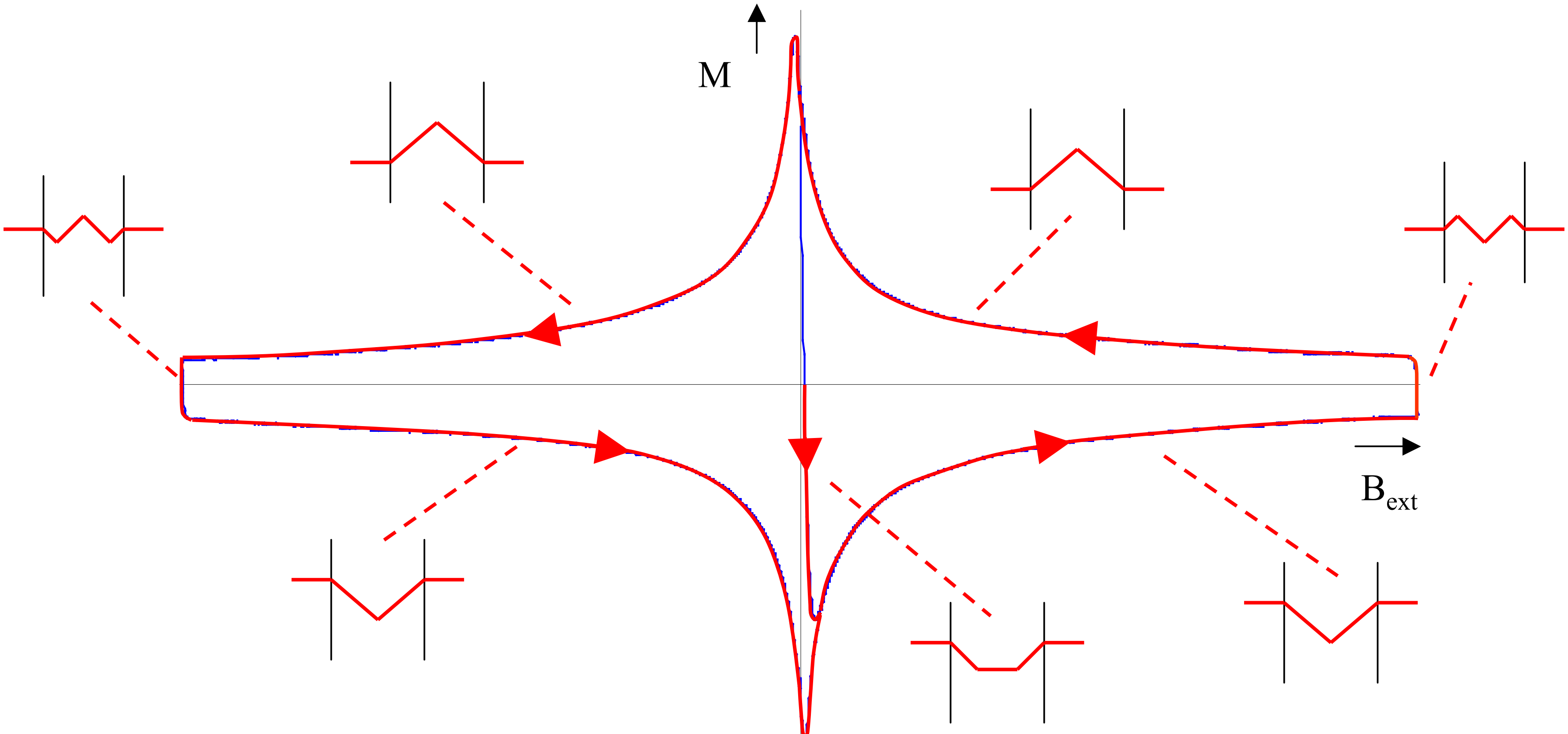


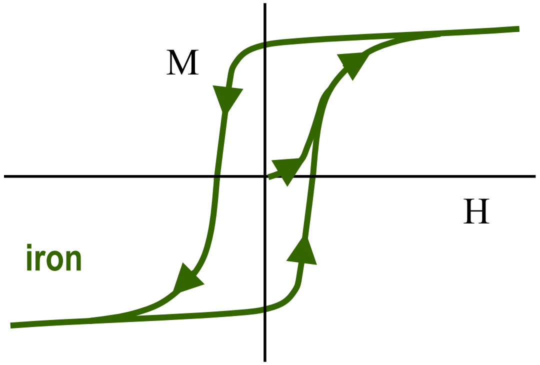
when fully penetrated, the magnetization is

$$M = \frac{4}{3\pi} J_c\, a$$

where a = filament radius
Note: M is here defined per unit volume of NbTi filament

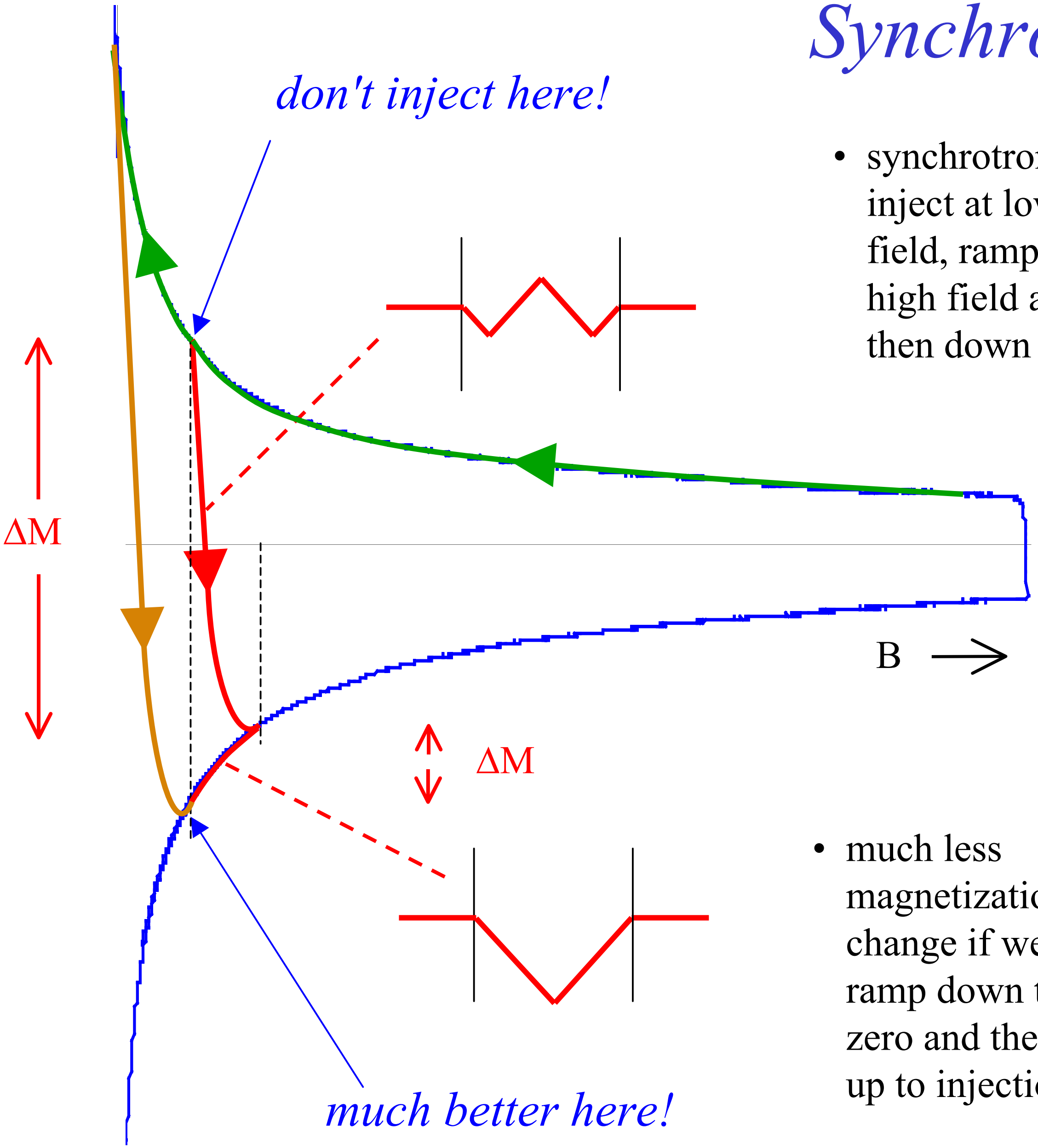
Magnetization of NbTi





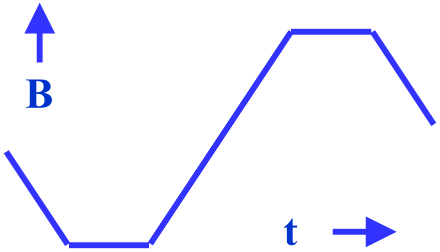
Magnetization is important because it produces field errors and ac losses

Synchrotron injection


don't inject here!

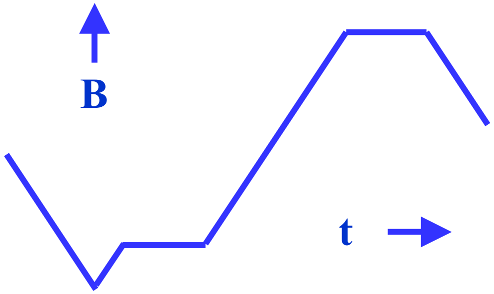
much better here!

- synchrotrons inject at low field, ramp to high field and then down again



- note how quickly the magnetization changes when we start the ramp up

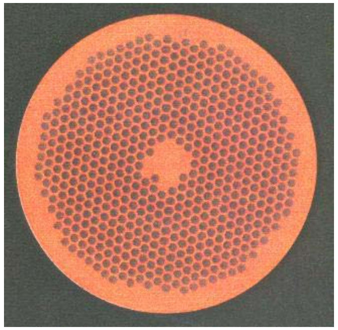
- much less magnetization change if we ramp down to zero and then up to injection

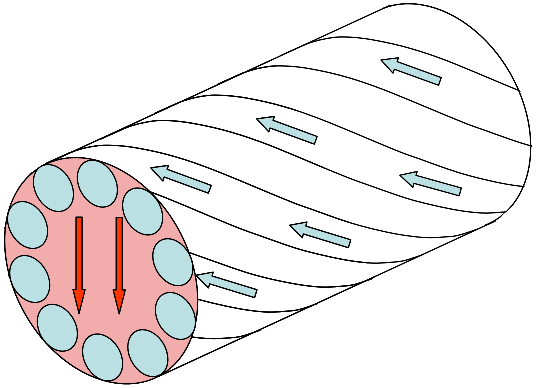


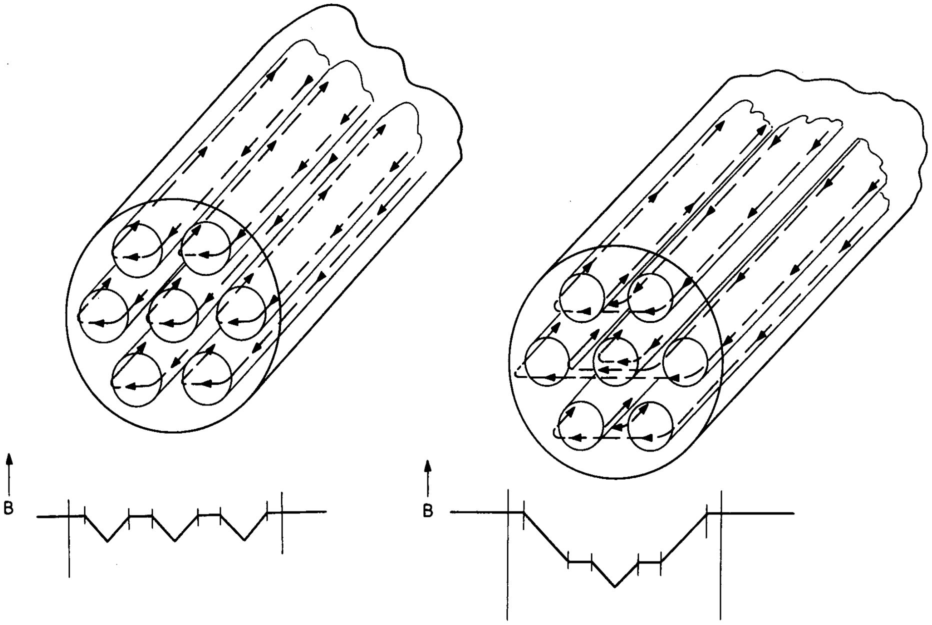
Coupling between filaments

recap $M = \frac{4}{3\pi} J_c a$

- reduce M by making fine filaments
- for ease of handling, filaments are embedded in a copper matrix







- but in changing fields, the filaments are magnetically coupled
- screening currents go up the left filaments and return down the right

- coupling currents flow along the filaments and across the matrix
- fortunately they may be reduced by twisting the wire
- they behave like eddy currents and produce an additional magnetization

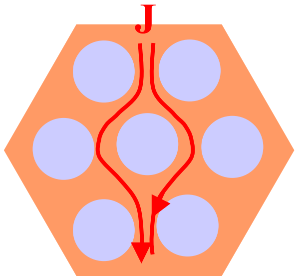
$$M_e = \frac{dB}{dt}\frac{1}{\rho_t}\left[\frac{p_w}{2\pi}\right]^2$$

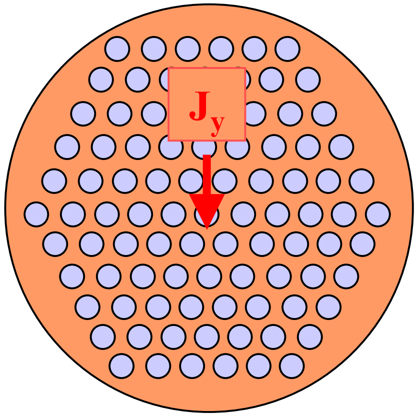
where ρ_t = resistivity across the matrix
and p_w = wire twist pitch

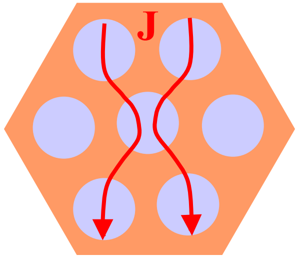
Transverse resistivity across the matrix

Poor contact to filaments

Good contact to filaments







$$\rho_t = \rho_{Cu} \frac{1+\lambda}{1-\lambda}$$

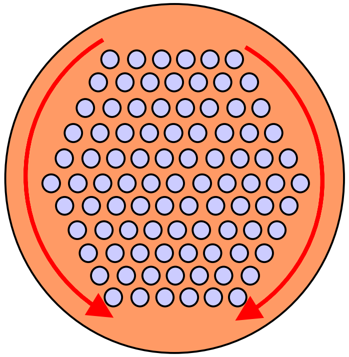
where λ is the fraction of superconductor in the wire cross section (after J Carr)

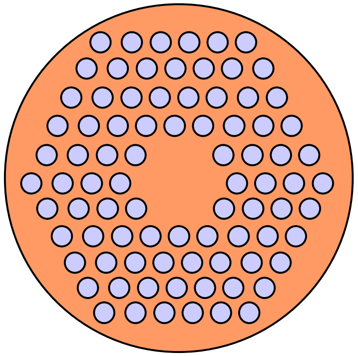
$$\rho_t = \rho_{Cu} \frac{1-\lambda}{1+\lambda}$$

Some complications

Thick copper jacket

include the
copper jacket
as a resistance
in parallel

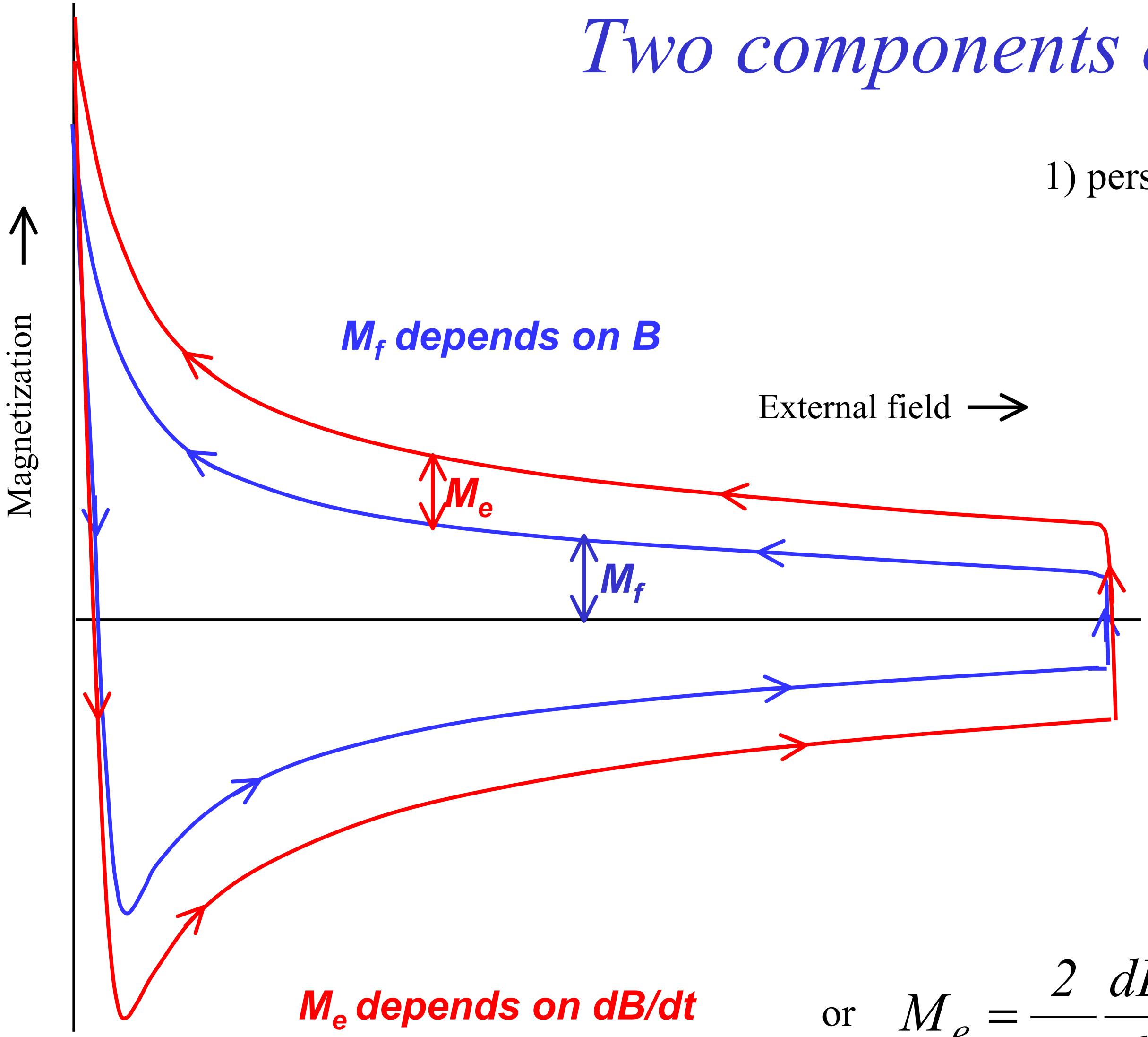




Copper core

resistance in
series for part
of current
path

Two components of magnetization



1) persistent current within the filaments

$$M_f = \frac{2}{3\pi} J_c(B) d_f$$

2) eddy current coupling between the filaments

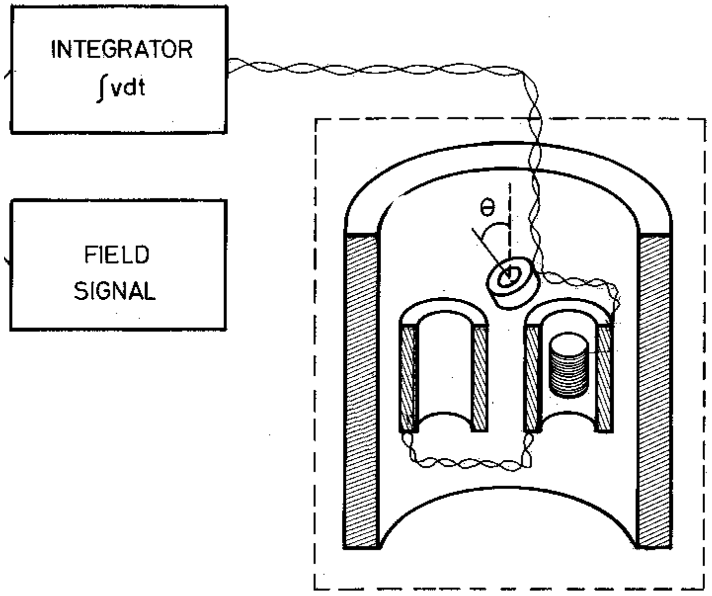
$$M_e = \frac{dB}{dt}\frac{1}{\rho_t}\left[\frac{p_w}{2\pi}\right]^2$$

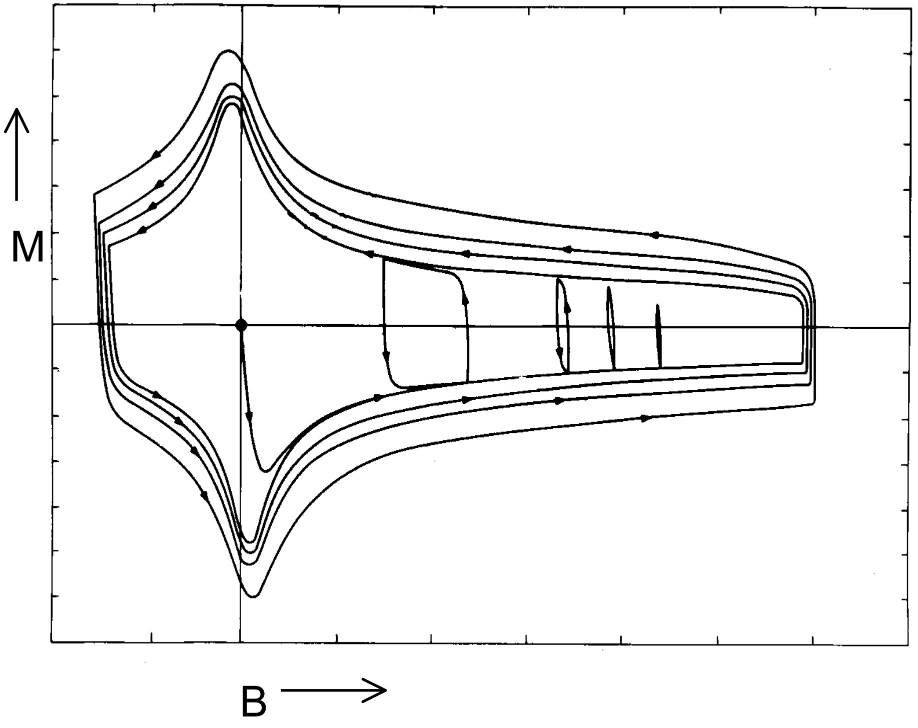
or $$M_e = \frac{2}{\mu_o}\frac{dB}{dt}\tau$$ where $$\tau = \frac{\mu_o}{2\rho_t}\left[\frac{p_w}{2\pi}\right]^2$$

Note M_f defined per unit volume of NbTi filament and M_e per unit volume of wire

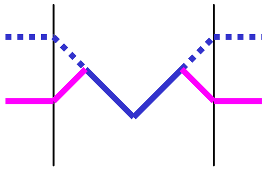
Measurement of magnetization

In field, the superconductor behaves just like a magnetic material. We can plot the magnetization curve using a magnetometer. It shows hysteresis - just like iron only in this case the magnetization is both diamagnetic and paramagnetic.



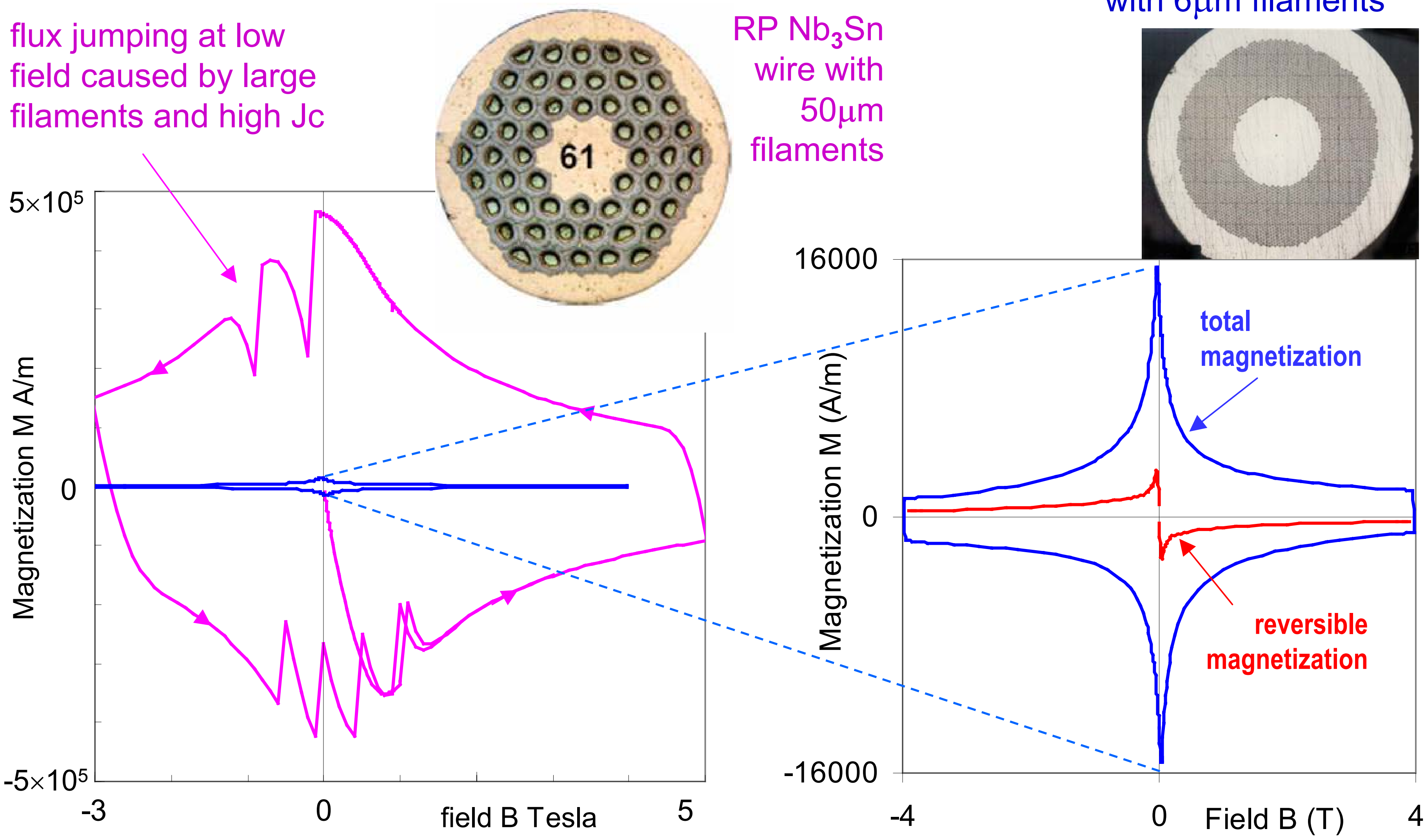


Note the minor loops, where field and therefore screening currents are reversing



The magnetometer, comprising 2 balanced search coils, is placed within the bore of a superconducting solenoid. These coils are connected in series opposition and the angle of small balancing coil is adjusted such that, with nothing in the coils, there is no signal at the integrator. With a superconducting sample in one coil, the integrator measures magnetization when the solenoid field is swept up and down

Magnetization measurements



flux jumping at low field caused by large filaments and high Jc

RP Nb_3Sn wire with 50μm filaments

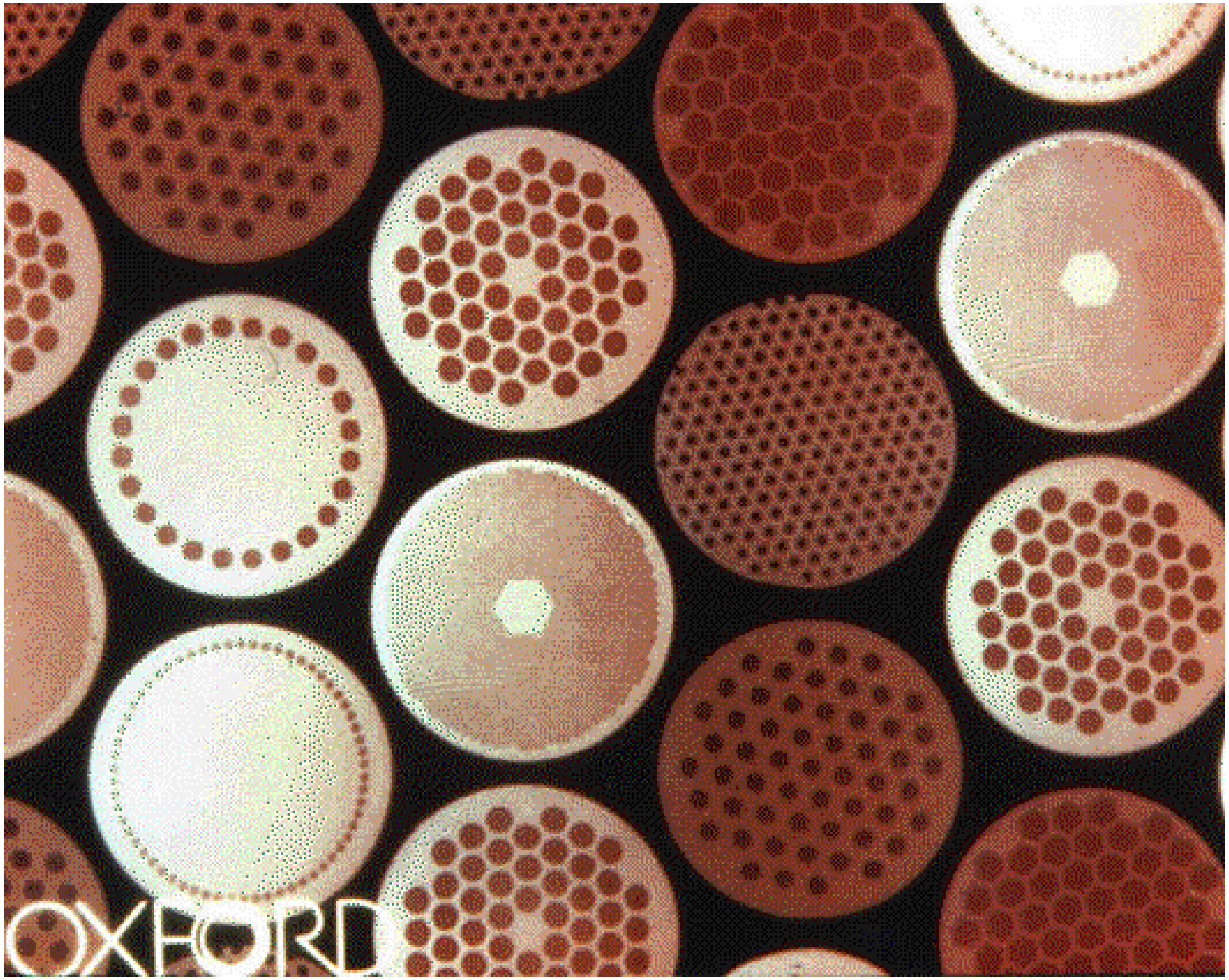
NbTi wire for RHIC with 6μm filaments

total magnetization

reversible magnetization

Fine filaments for low magnetization

- the finest filaments are made for accelerator magnets, mainly to keep the field errors at injection down to an acceptable level.
- typical diameters are in the range 5 - 10μm. Even smaller diameters would give lower magnetization, but at the cost of lower Jc and more difficult production.



Cables - why do we need them?

- for good tracking we connect synchrotron magnets in series
- if the stored energy is E, rise time t and operating current I, the charging voltage is

$$E = \frac{1}{2} L I^2 \qquad V = \frac{L I}{t} = \frac{2E}{I t}$$

RHIC E = 40kJ/m, t = 75s, 30 strand cable

cable I = **5kA**, charge voltage per km = **213V**

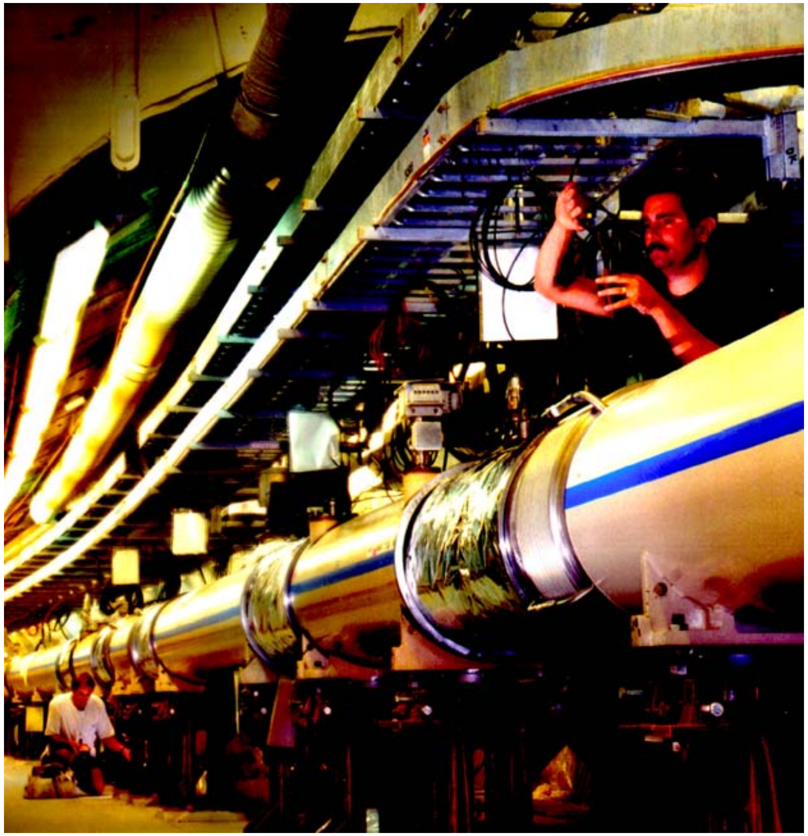
wire I = **167A**, charge voltage per km = **6400V**

FAIR at GSI E = 74kJ/m, t = 4s, 30 strand cable

cable I = **6.8kA**, charge voltage per km = **5.4kV**

wire I = **227A**, charge voltage per km = **163kV**

- so we need high currents!

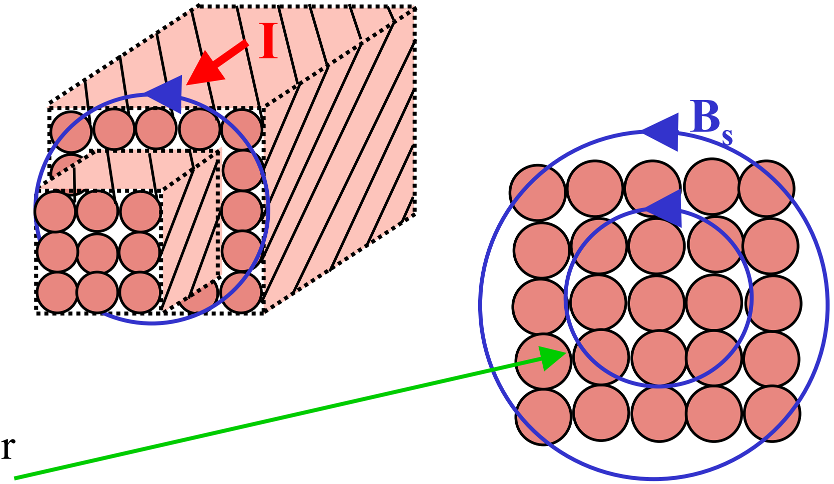


the RHIC tunnel

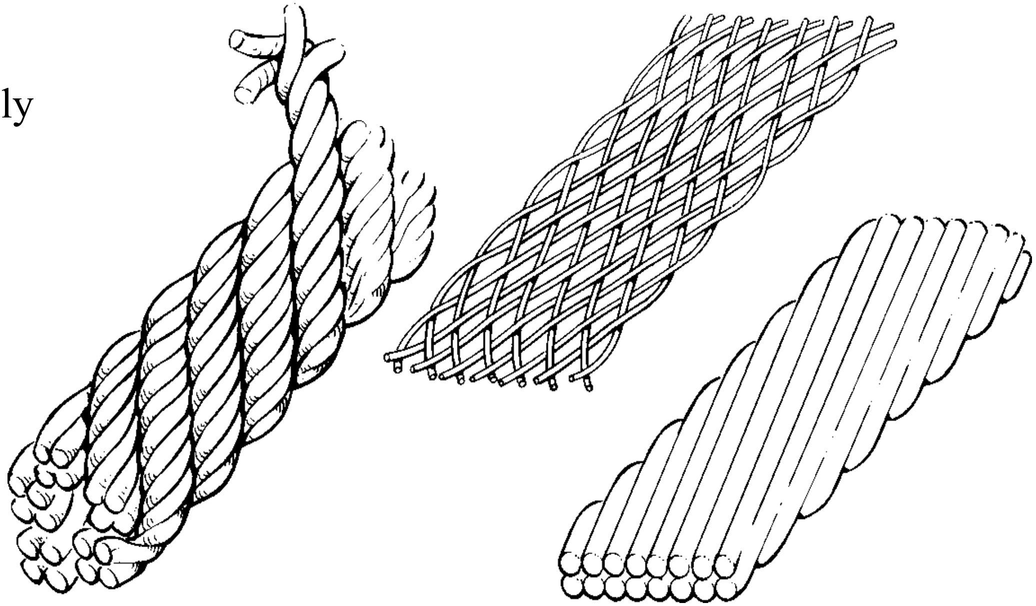
- a single 5µm filament of NbTi in 6T carries 50mA
- a composite wire of fine filaments typically has 5,000 to 10,000 filaments, so it carries 250A to 500A
- for 5 to 10kA, we need 20 to 40 wires in parallel - a cable

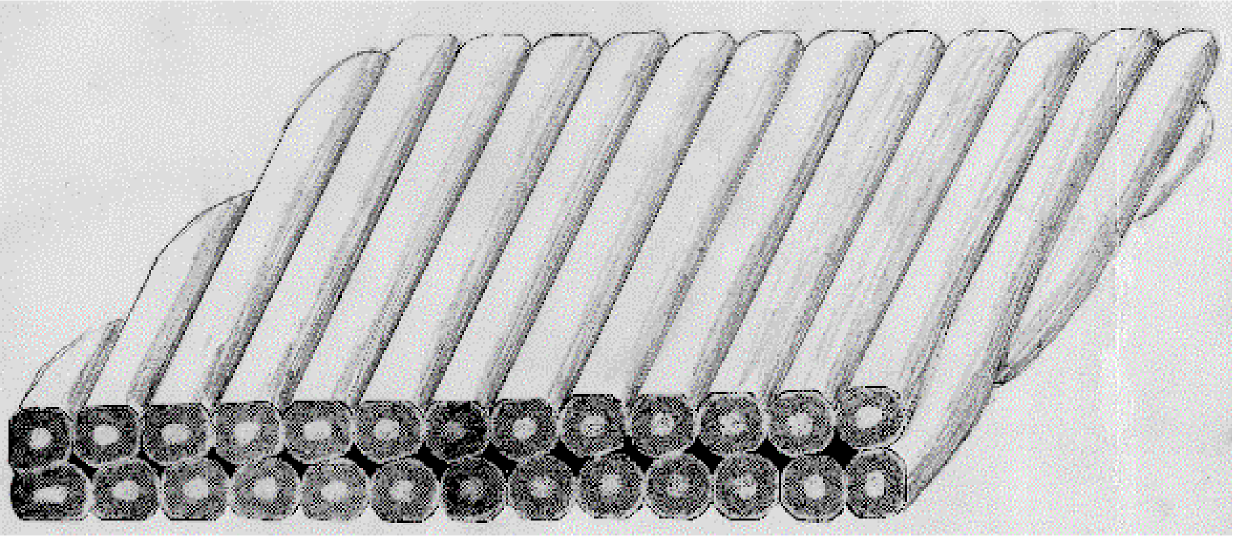
Types of cable

- cables carry a large current and this generates a **self field**
- in this cable the self field generates a flux between the inner and outer wires ⇒
- wire are twisted to avoid flux linkage between the filaments, for the same reasons we should avoid flux linkage between wires in a cable
- but twisting this cable doesn't help because the inner wires are always inside and the outers outside



- thus it is necessary for the wires to be fully **transposed**, ie every wire must change places with every other wire along the length of the cable so that, averaged over the length, no flux is enclosed
- three types of fully transposed cable have been tried in accelerators
  - rope
  - braid
  - Rutherford



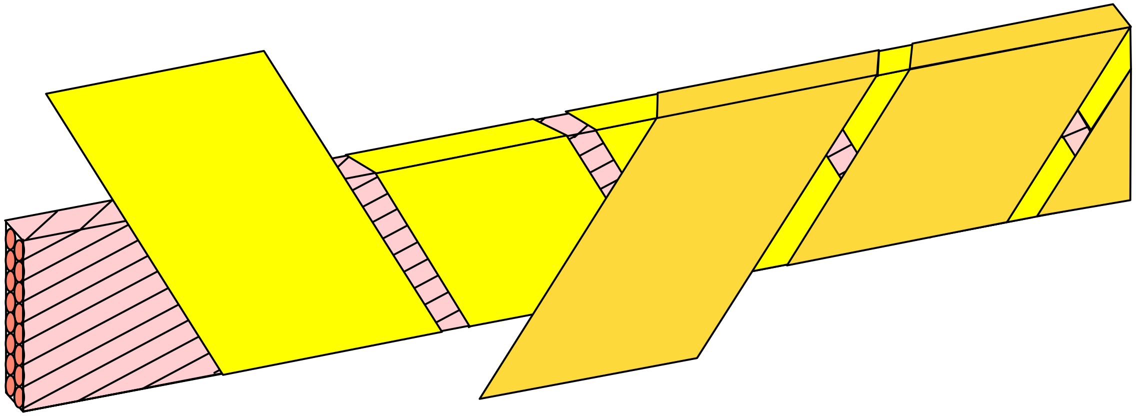


Rutherford cable

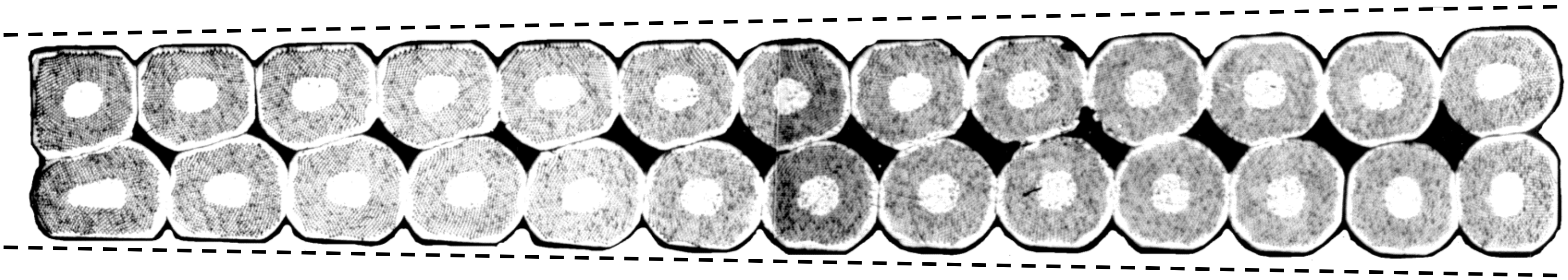
- the cable is insulated by wrapping 2 or 3 layers of Kapton; gaps may be left to allow penetration of liquid helium; the outer layer is treated with an adhesive layer for bonding to adjacent turns.

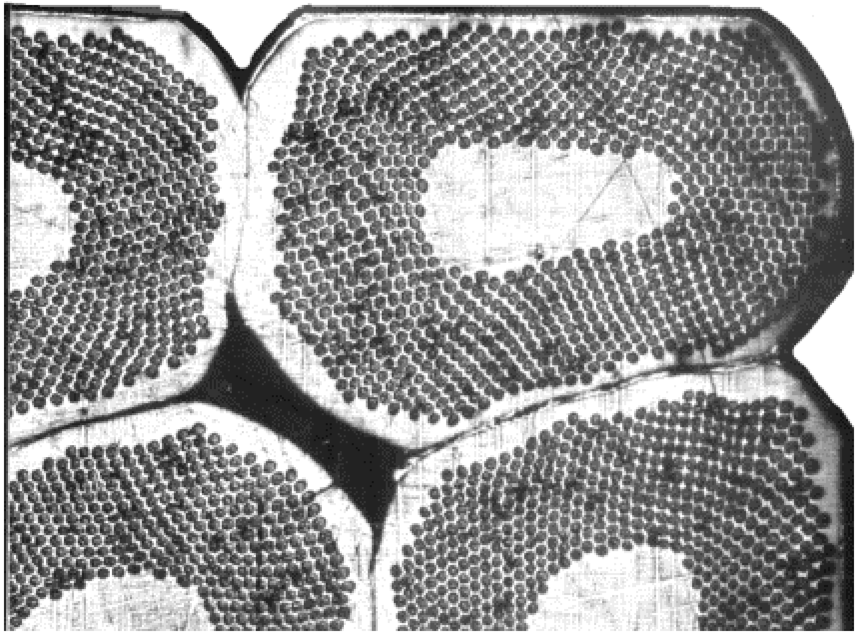
- Recapitulate: the adhesive faces outwards, don't bond it to the cable (avoid energy release by bond failure)

- allow liquid helium to permeate the cable
  - increase the MQE



Rutherford cable

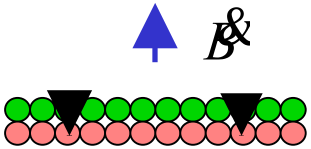




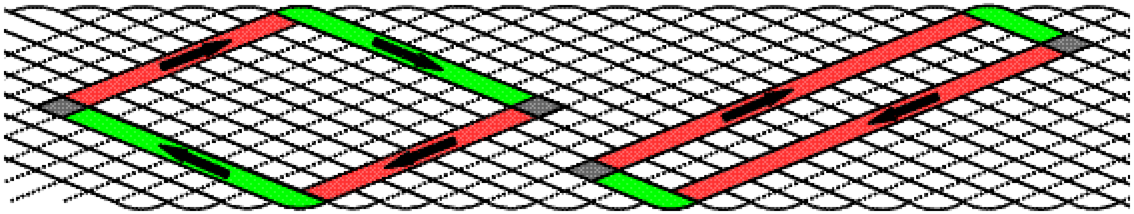
- The main reason why Rutherford cable succeeded where others failed was that it could be compacted to a high density (88 - 94%) without damaging the wires. Furthermore it can be rolled to a good dimensional accuracy (~ 10mm).
- Note the 'keystone angle', which enables the cables to be stacked closely round a circular aperture

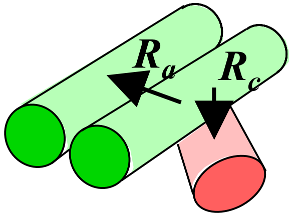
Coupling in Rutherford cables

- Field transverse
  coupling via crossover resistance R_c

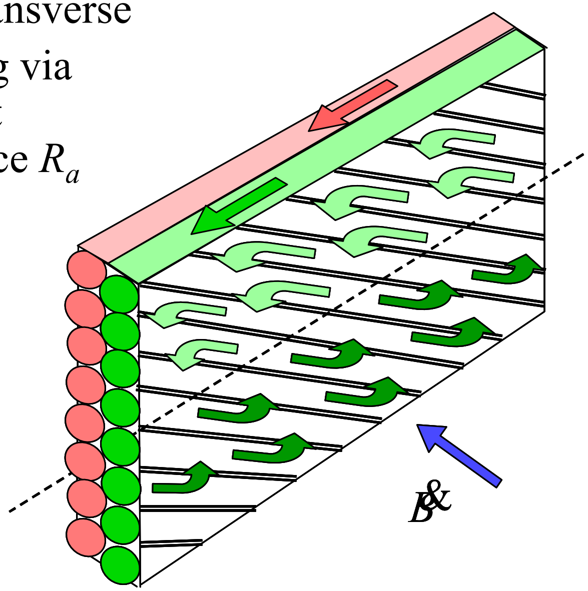


crossover resistance *Rc*
adjacent resistance *Ra*

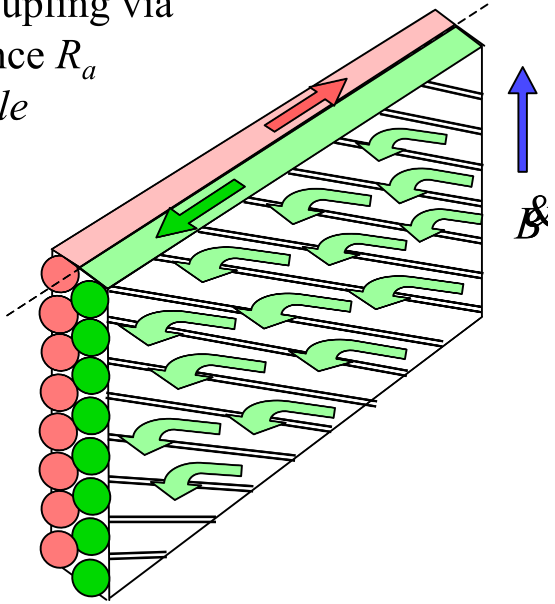




- Field transverse coupling via adjacent resistance R_a



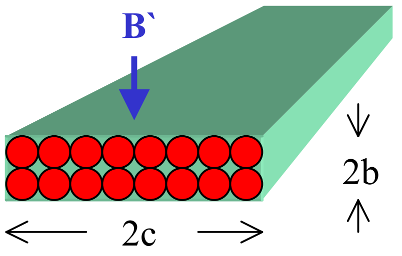
- Field parallel coupling via adjacent resistance R_a
  usually negligible



Magnetization from coupling in cables

- Field transverse coupling via crossover resistance R_c

$$M_{tc} = \frac{1}{120}\frac{\dot{B}_t}{R_c}\frac{c}{b}\,p\,N(N-1) = \frac{1}{60}\frac{\dot{B}_t}{r_c}\,p^2\,\frac{c^2}{b}$$



where M = magnetization per unit volume of cable, p= twist pitch, N = number of strands
R_c R_a resistance per crossover r_c r_a resistance per unit area of contact

- Field transverse coupling via adjacent resistance R_a

  where θ = slope angle of wires $Cos\theta$ ~ 1

$$M_{ta} = \frac{1}{6}\frac{\dot{B}_t}{R_a}\,p\,\frac{c}{b} = \frac{1}{48}\frac{\dot{B}_t}{r_a}\,p^2\,\frac{b}{Cos^2\theta}$$

- Field parallel coupling via adjacent resistance R_a

$$M_{pa} = \frac{1}{8}\frac{\dot{B}_p}{R_a}\,p\,\frac{b}{c} = \frac{1}{64}\frac{\dot{B}_p}{r_a}\,p^2\,\frac{b^3}{c^2\cos^2\theta}$$

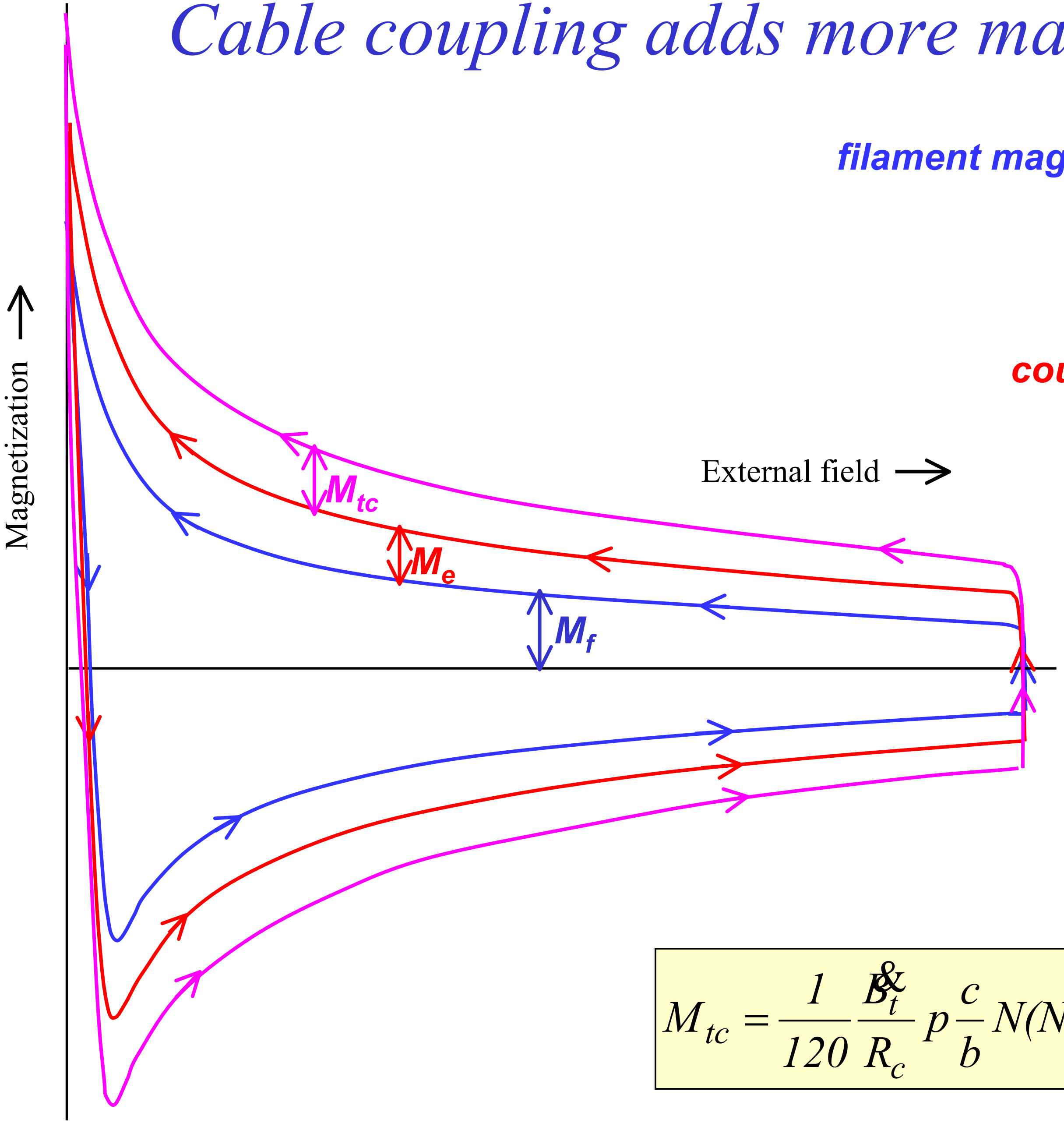
(usually negligible)

- Field transverse ratio crossover/adjacent

$$\frac{M_{tc}}{M_{ta}} = \frac{R_a}{R_c}\frac{N(N-1)}{20} \approx 45\frac{R_a}{R_c}$$

So without increasing loss too much can make $\boldsymbol{R_a}$ 50 times less than $\boldsymbol{R_c}$ - anisotropy

Cable coupling adds more magnetization



filament magnetization M_f depends on B

$$M_f = \frac{2}{3\pi} J_c(B) d_f$$

coupling between filaments M_e depends on dB/dt

$$M_e = \frac{dB}{dt}\frac{1}{\rho_t}\left[\frac{p_w}{2\pi}\right]^2$$

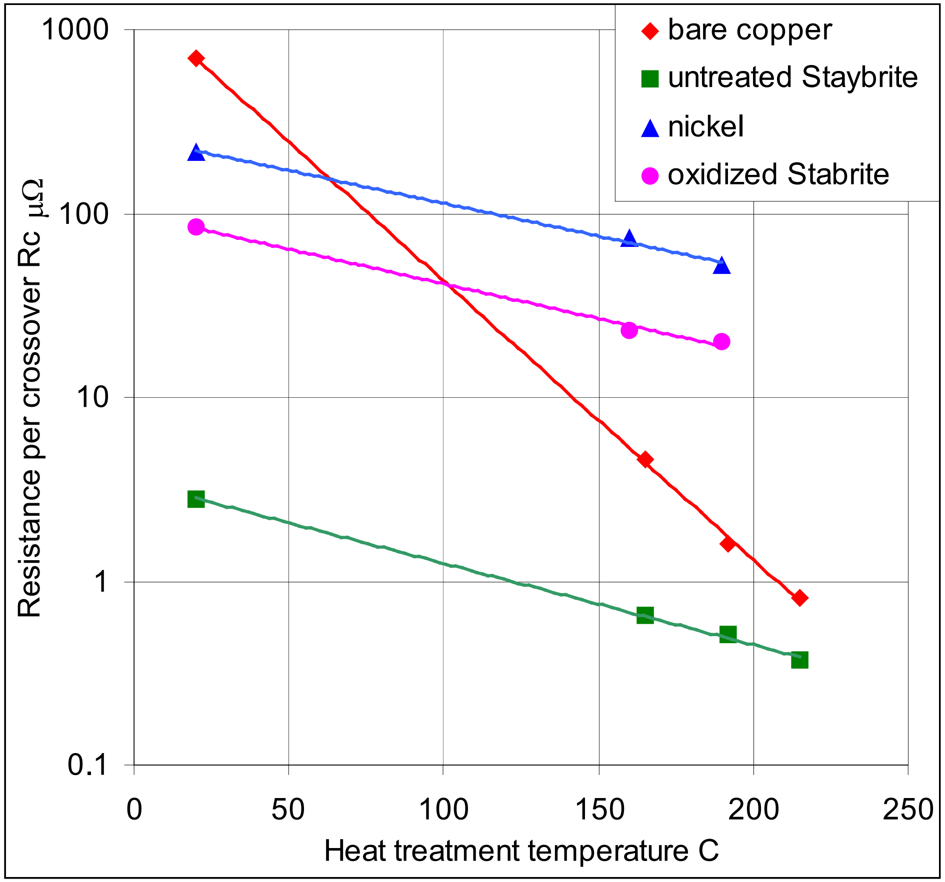
coupling between wires in cable depends on dB/dt

$$\boxed{M_{tc} = \frac{1}{120}\frac{\dot{B}_t}{R_c} p \frac{c}{b} N(N-1)}$$

$$\boxed{M_{ta} = \frac{1}{6}\frac{\dot{B}_t}{R_a} p \frac{c}{b}}$$

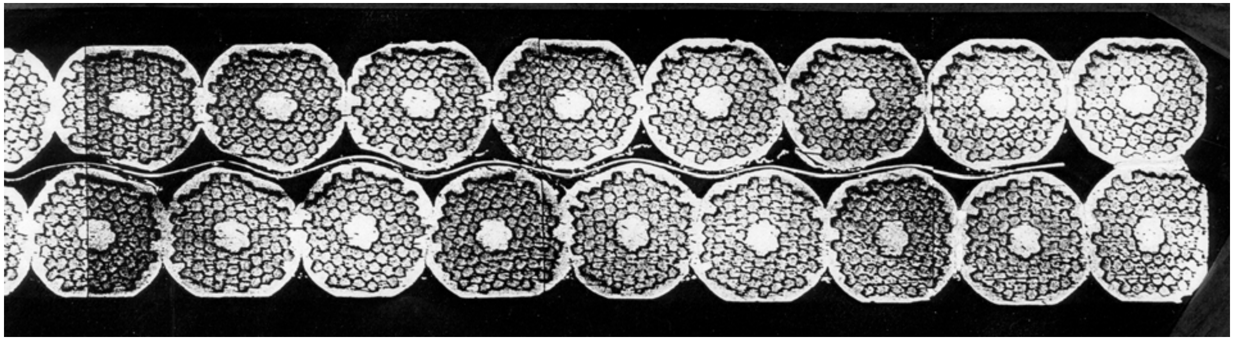
Controlling R_a and R_c

- surface coatings on the wires are used to adjust the contact resistance
- the values obtained are very sensitive to pressure and heat treatments used in coil manufacture (to cure the adhesive between turns)
- *data from David Richter CERN*



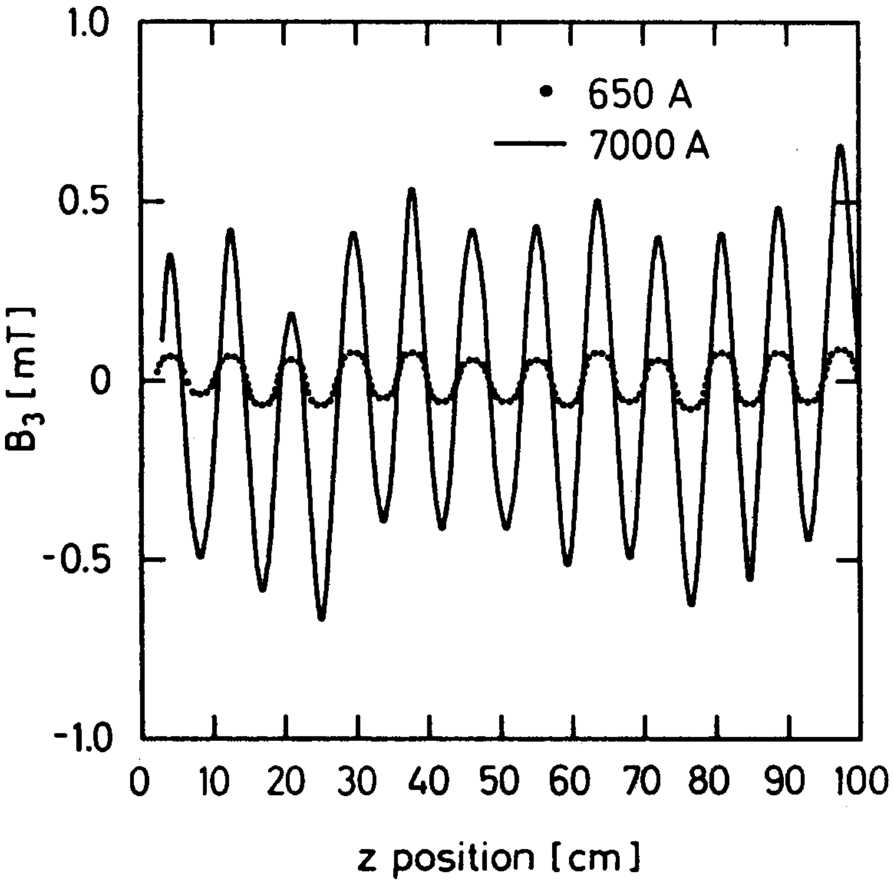
Cored Cables

- using a resistive core allows us to increase R_c while keeping R_a the same
- thus we reduce losses but still maintain good current transfer between wires
- not affected by heat treatment



Long range coupling: BICCs

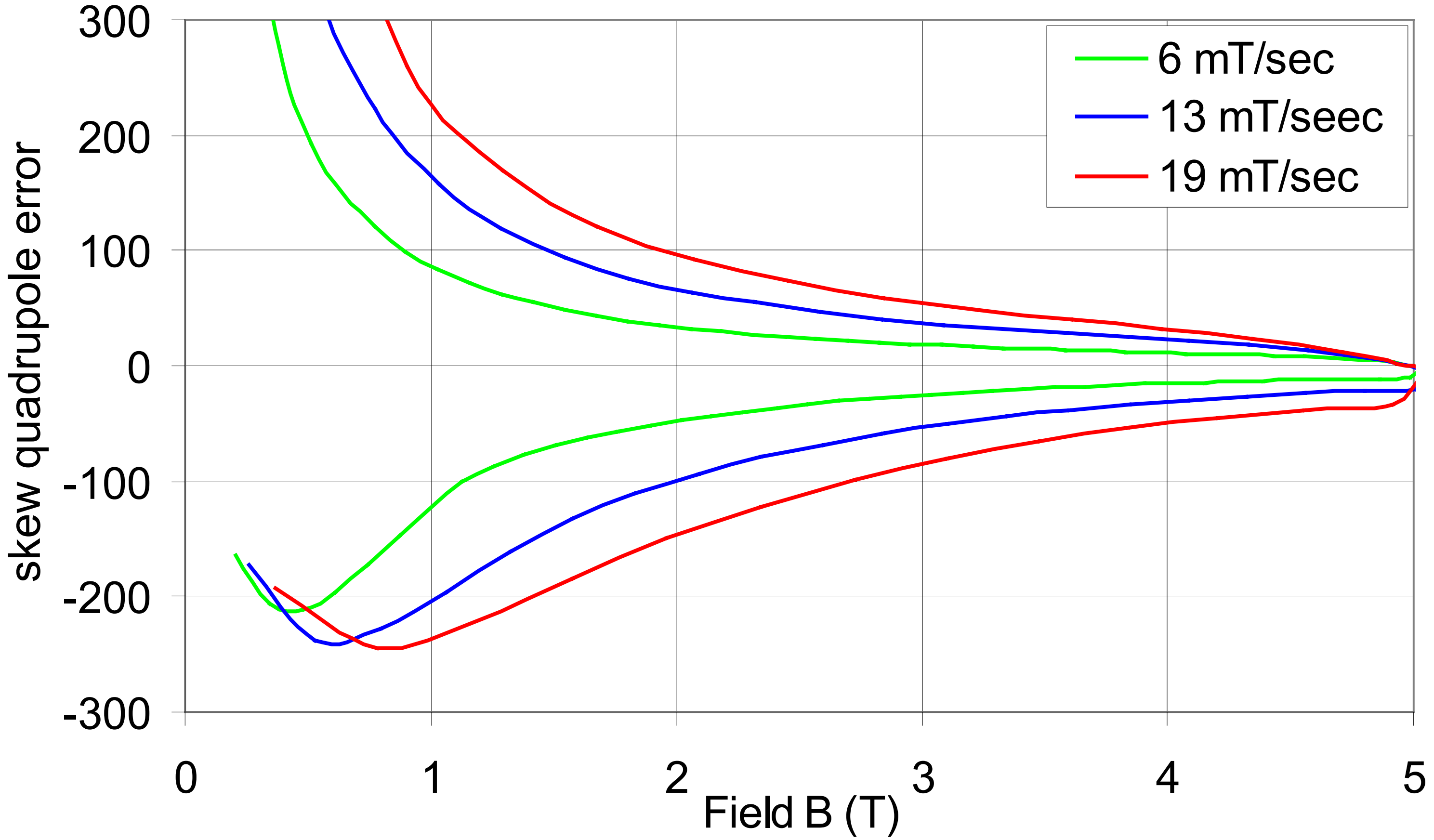
- measuring the field of an accelerator magnet along the beam direction, we find a ripple
- wavelength of this ripple exactly matches the twist pitch of the cable
- thought to be caused by non uniform current sharing in the cable
- Verweij has called them 'boundary induced coupling currents' **BICCs**
- they are caused by non uniform flux linkages or resistances in the cable, eg at joints, coil ends, manufacturing errors etc.
- wavelength is << betatron wavelength so no direct problem, but interesting secondary effects such as **'snap back'.**



sextupole measured in SSC dipole at injection and full field

Magnetization and field errors - extreme case

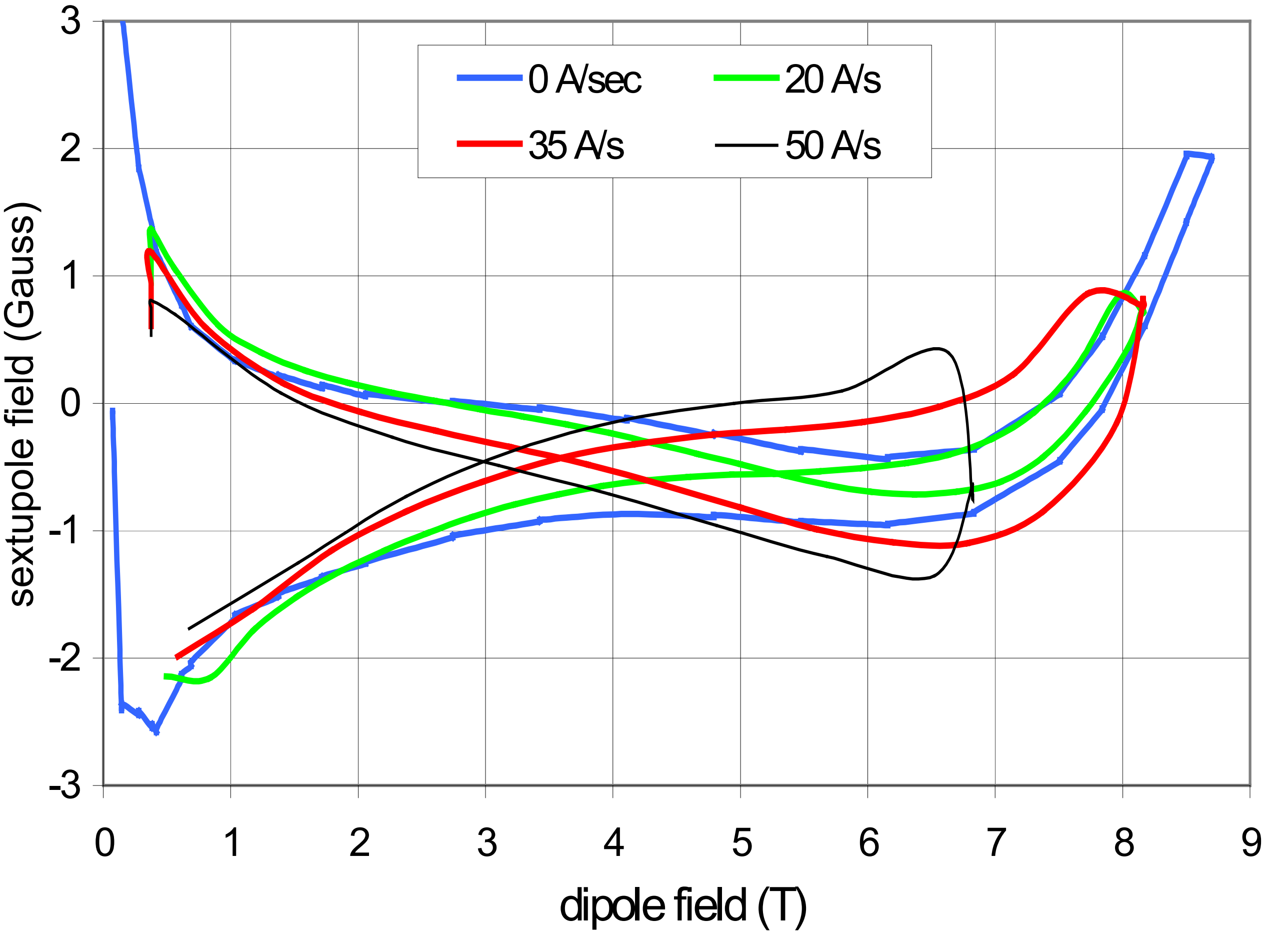
Magnetization is important in accelerators because it produces field error. The effect is worst at injection because
- $\Delta B/B$ is greatest
- magnetization, ie ΔB is greatest at low field



skew quadrupole error in Nb$_3$Sn dipole which has exceptionally large coupling magnetization (University of Twente)

Magnetization and field errors - quirky case

- plot of sextupole field error in an LHC dipole with field ramped at different rates
- error at low field due to filament magnetization
- error at high field due to
  a) iron saturation
  b) coupling between strands of the cable
- the curves turn 'inside out' because
  - greatest **filament** magnetization is in the **low** field region (high J_c)
  - greatest **coupling** is in the **high** field region (high dB/dt)



data from Luca Bottura CERN

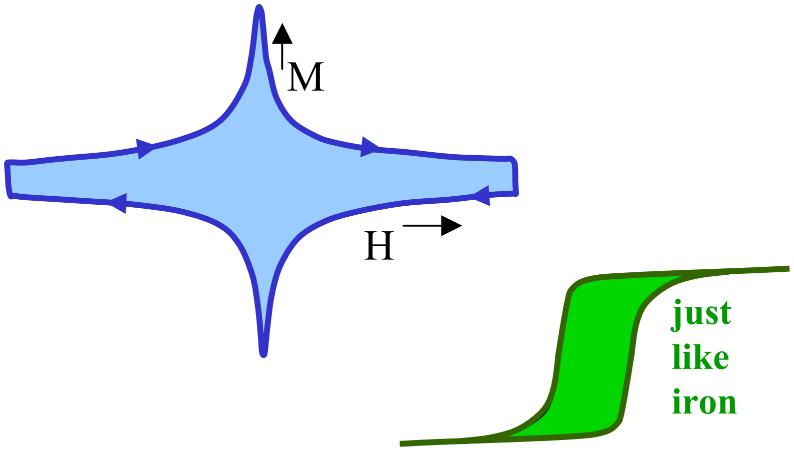
AC Losses

Physics viewpoint

the change in magnetic field energy

$$\delta E = H\delta B$$

(see textbooks on electromagnetism)



so work done on magnetic material

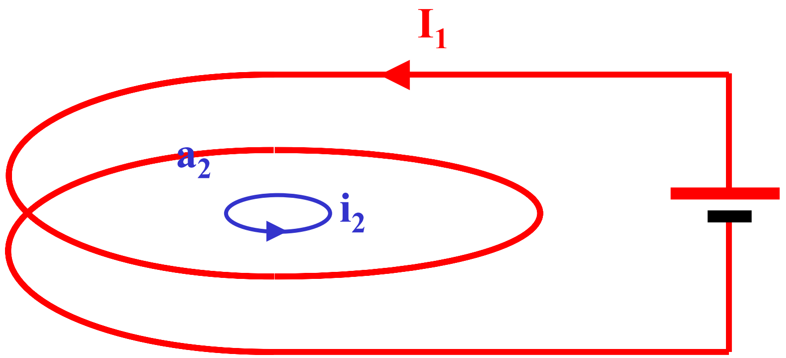
$$W = \int \mu_o H dM$$

around a ***closed loop***, this integral must be the energy dissipated in the material

$$E = \int \mu_o H dM = \int \mu_o M dH$$

Engineering viewpoint

element of magnetization represented by current loop $\mathbf{i_2}$



work done by battery to raise current $\mathbf{I_1}$ in solenoid

$$W = \int V_1 I_1 dt = \int I_1 L_{11} \frac{dI_1}{dt} dt - \int I_1 L_{21} \frac{di_2}{dt} dt$$

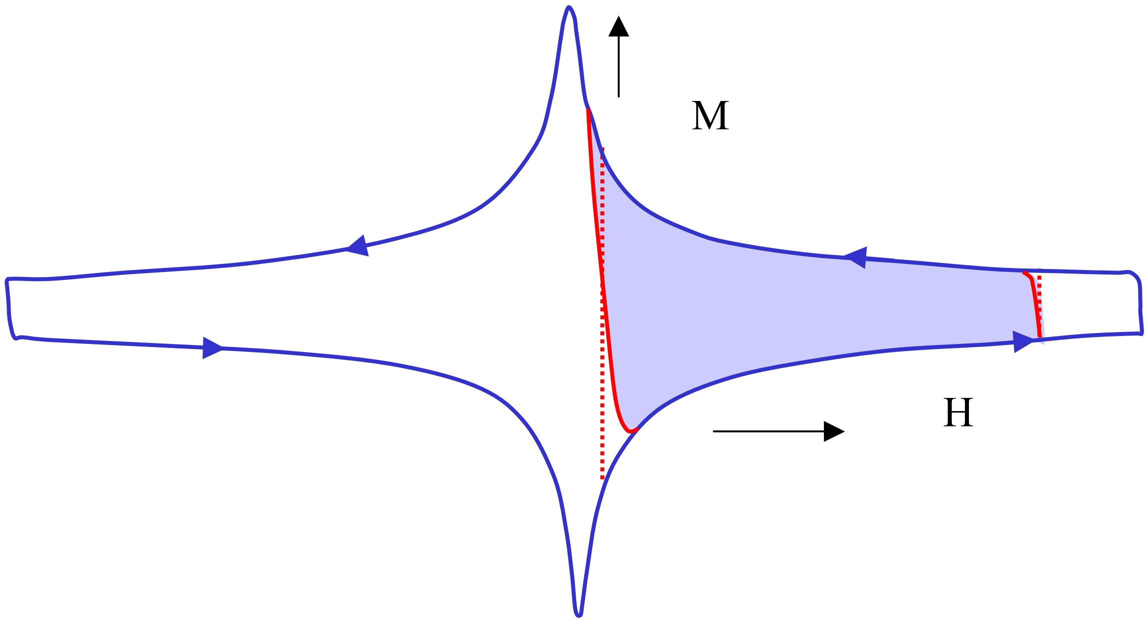
$$= \frac{1}{2} L_{11} I_1^2 - \int I_1 L_{21} di_2$$

first term is change in stored energy of solenoid
$I_1 L_{21}$ is the flux change produced in loop **2**

$$\int I_1 L_{21} di_2 = \int \mu_o H_1 a_2 di_2 = \int \mu_o H_1 dM$$

so work done on loop by battery $= \int \mu_o H_1 dM$

Loss Power



$$W = \int \mu_o H dM = \int \mu_o M dH$$

This is the work done on the sample
Strictly speaking, we can only say it is a heat dissipation if we integrate round a loop and come back to the same place
- otherwise the energy might just be stored

Around a loop the red 'crossover' sections are complicated, but we usually approximate them as straight vertical lines (dashed)

With the approximation of vertical lines at the **'turn around points'** and saturation magnetization in between, the hysteresis loss per cycle is

$$E = \oint \mu_o M dH \cong \oint M dB$$

In the (usual) situation where $dH>>M$, we may write the loss between two fields B_1 and B_2 as

$$E \cong \int_{B_1}^{B_2} M dB$$

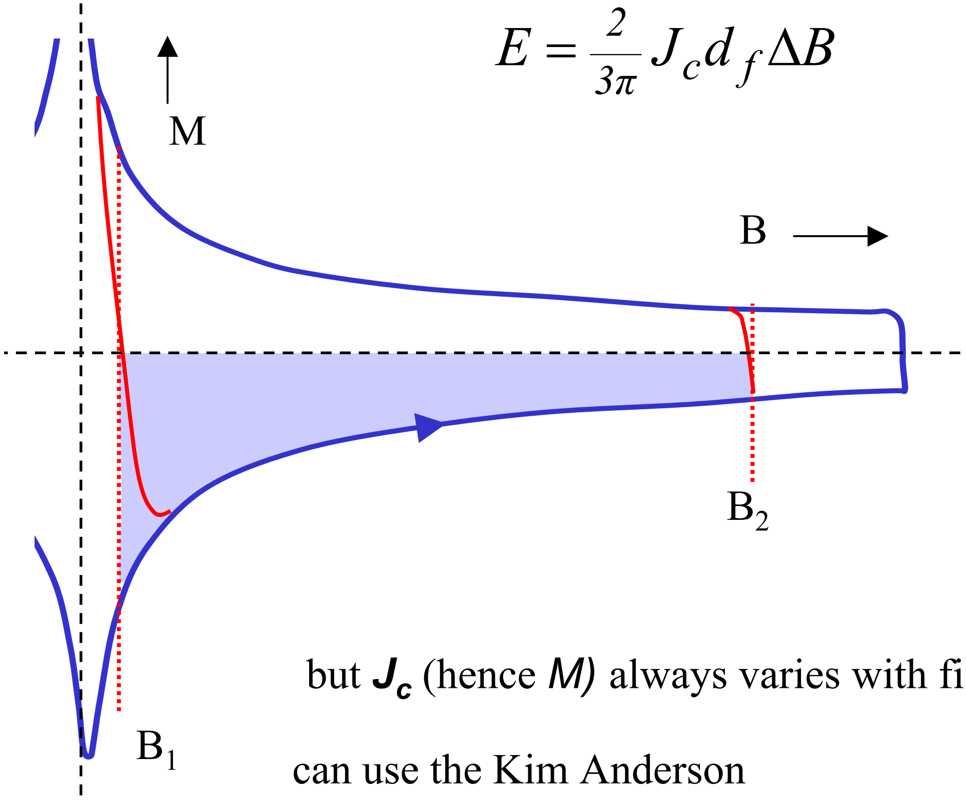
so the loss power is $\boxed{P = M\dot{B}}$

M in A.m^{-1}, B in Tesla, losses in Joules.m^{-3} and Watts.m^{-3} of superconductor

Hysteresis loss within in the superconducting filaments

with constant $\boldsymbol{J_c}$ $$M = \frac{2}{3\pi} J_c\, d_f$$

$$E = \frac{2}{3\pi} J_c d_f \Delta B$$



but $\boldsymbol{J_c}$ (hence *M)* always varies with field

can use the Kim Anderson approximation

$$J_c(B) = \frac{J_o B_o}{(B + B_o)}$$

so $$M = \frac{2}{3\pi} d_f \frac{J_o B_o}{(B + B_o)}$$

$$E \cong \int_{B_1}^{B_2} M dB$$

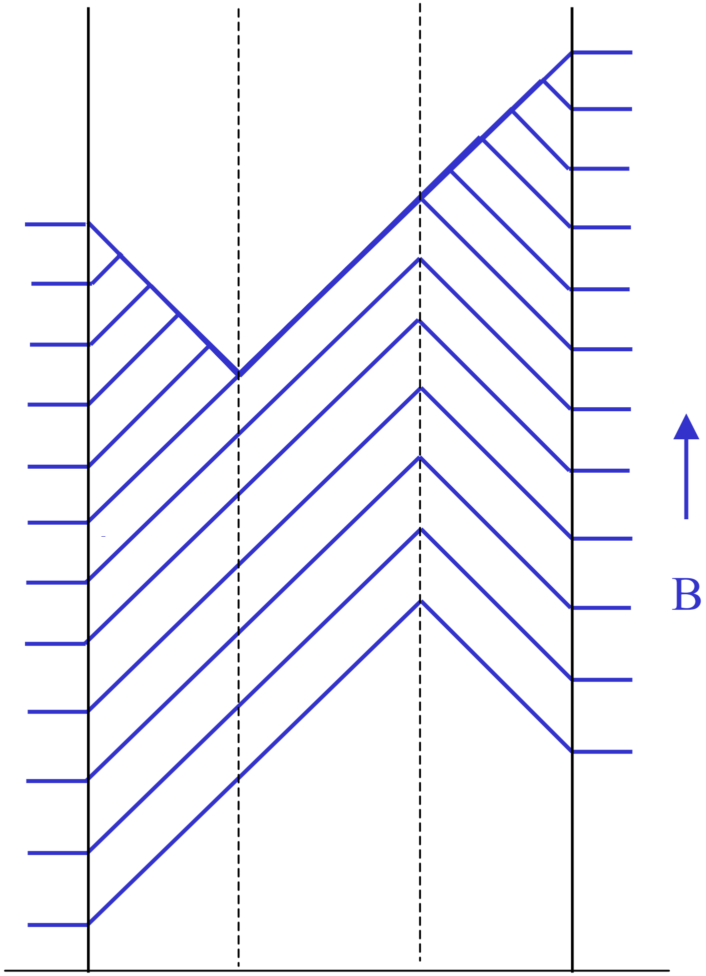
loss for ramp up from $\boldsymbol{B_1}$ to $\boldsymbol{B_2}$

$$E = \frac{2}{3\pi} \int_{B_1}^{B_2} \frac{J_o B_o}{(B + B_o)} d_f dB$$

$$\boxed{E = \frac{2}{3\pi} d_f J_o B_o \ln\left\{\frac{B_2 + B_o}{B_1 + B_o}\right\}}$$

loss in Joules per m^3 of superconductor

The effect of transport current



plot field profile across the slab

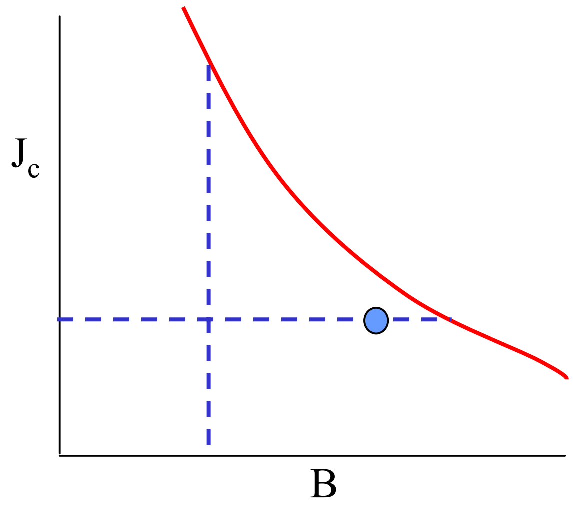
- in magnets there is a transport current, coming from the power supply, in addition to magnetization currents.
- because the transport current 'uses up' some of the available $\boldsymbol{J_c}$ the magnetization is reduced.
- but the loss is increased because the power supply does work and this adds to the work done by external field

total loss is increased by factor $(1+\boldsymbol{i}^2)$ where $\boldsymbol{i} = \boldsymbol{I_{max}} / \boldsymbol{I_c}$

$$\boxed{E = \frac{2}{3\pi} d_f J_o B_o \ln\left\{\frac{B_2 + B_o}{B_1 + B_o}\right\}(1+i^2)}$$

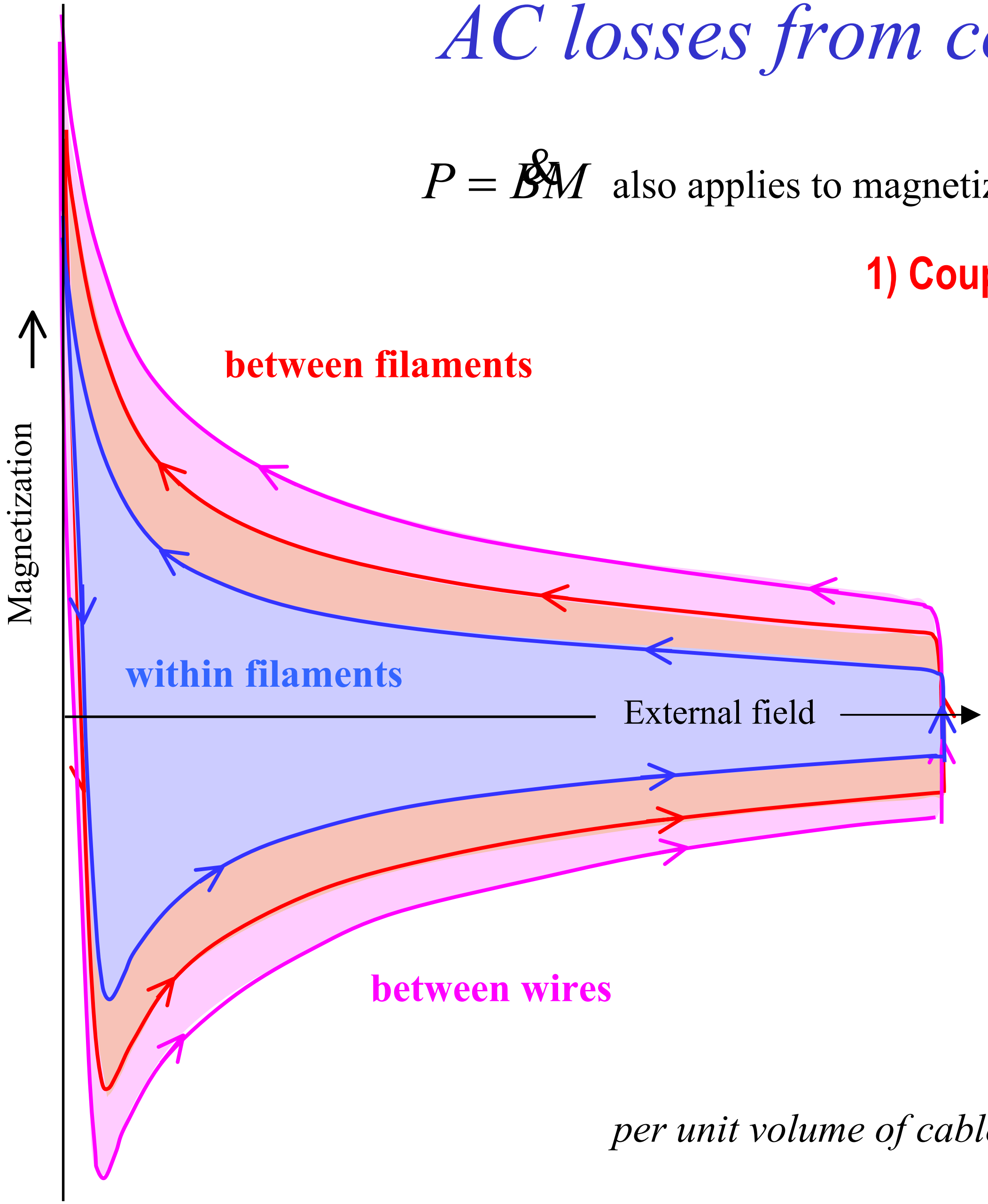
usually not such a big factor because

- *design for a margin in Jc*
- *most of magnet is in a field much lower than the peak*



AC losses from coupling

$P = \dot{B}M$ also applies to magnetization coming from coupling



1) Coupling between filaments within the wire

$$P_e = \dot{B}^2 \frac{1}{\rho_t}\left[\frac{p_w}{2\pi}\right]^2$$

per unit volume of wire

2) Coupling between wires in the cable

$$P_{tc} = \frac{1}{120}\frac{\dot{B}_t^2}{R_c}\frac{c}{b}p_c\,N(N-1)$$

$$P_{ta} = \frac{1}{6}\frac{\dot{B}_t^2}{R_a}p_c\frac{c}{b}$$

$$P_{pa} = \frac{1}{8}\frac{\dot{B}_p^2}{R_a}p_c\frac{b}{c}$$

per unit volume of cable

Summary of losses - per unit volume of winding

1) Persistent currents in filaments

power W.m^{-3}

$$P_f = \lambda_c \lambda_w \lambda_f M_f \dot{B} = \lambda_c \lambda_w \lambda_f \frac{2}{3\pi} J_c(B)\, d_f \dot{B}$$

loss per per ramp J.m-3

$$E_f = \lambda_c \lambda_w \lambda_f \frac{2}{3\pi} d_f J_o B_o \ln\left\{\frac{B_2 + B_o}{B_1 + B_o}\right\}$$

where λ_f = fraction of filament in wire, λ_w = fraction of wire in cable, λ_c = fraction of cable in winding

2) Coupling currents between filaments in the wire

power W.m^{-3}

$$P_e = \lambda_c \lambda_w \lambda_{fb} M_e \dot{B} = \lambda_c \lambda_w \lambda_{fb} \frac{\dot{B}^2}{\rho_t}\left(\frac{p}{2\pi}\right)^2$$

3) Coupling currents between wires in the cable

transverse field crossover resistance power W.m^{-3}

$$P_{tc} = \lambda_c \frac{1}{120} \frac{\dot{B}_t^2}{R_c} p \frac{c}{b} N(N-1)$$

transverse field adjacent resistance power W.m^{-3}

$$P_{ta} = \lambda_c \frac{1}{6} \frac{\dot{B}_t^2}{R_a} p \frac{c}{b}$$

parallel field adjacent resistance power W.m^{-3}

$$P_{pa} = \lambda_c \frac{1}{8} \frac{\dot{B}_p^2}{R_a} p \frac{b}{c}$$

Concluding remarks

- screening currents produce magnetization (magnetic moment per unit volume)
  - ⇒ lots of problems **-** field errors and ac losses
- in a synchrotron, the field errors from magnetization are worst at injection
- we reduce magnetization by making fine filaments - for practical use embed them in a matrix
- in changing fields, filaments are coupled through the matrix ⇒ increased magnetization
  - reduce it by twisting and by increasing the transverse resistivity of the matrix
- accelerator magnets must run at high current because they are all connected in series
  - combine wires in a cable, it must be fully transposed to ensure equal currents in each wire
- wires in cable must have some resistive contact to allow current sharing
  - in changing fields the wires are coupled via the contact resistance
    - different coupling when the field is parallel and perpendicular to face of cable
      - coupling produces more magnetization ⇒ more field errors
- irreversible magnetization ⇒ ac losses in changing fields
  - coupling between filaments in the wire adds to the loss
    - coupling between wire in the cable adds more

never forget that magnetization and ac loss are defined per unit volume - ***filling factors***